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NONLINEAR EFFECT OF EXTERNAL LOW-FREQUENCY ACOUSTICS ON EIGEN-OSCILLATIONS IN A SUPERSONIC BOUNDARY LAYER

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A method of simulation and results of numerical calculation of the evolution of hydrodynamic disturbances in a supersonic boundary layer on a flat plate under the influence of external acoustic waves at Reynolds numbers Re = 220–640 *and Mach number* M = 2 *are described. The solution is constructed by the method of expansion with respect to the small parameter; the contribution of linear and quadratic terms to the solution is taken into account. The method developed allows one to estimate the admissible level of the acoustic field, which does not affect the development of eigen-oscillations in the boundary layer.*

Introduction. The problem of nonlinear interaction of acoustic waves and eigen-oscillations in a supersonic boundary layer is directly related to the problem of receptivity of steady flows to external actions; in the linear formulation, the latter problem involves determination of the amplitude of acoustic vibrations for a given magnitude of action. It should be emphasized that, if the main flow is parallel, external monochromatic waves do not excite eigen-oscillations [1]. The problem of excitation of eigen-oscillations by a monochromatic acoustic wave owing to nonparallelism of the main flow was considered in the linear formulation for the first time by Gaponov [2].

In the nonlinear formulation of the problem, the external wave can be considered as a pumping wave with eigen-oscillations developing in its field. An example of such a process is the development of disturbances in the boundary layer on a model located in the test section of a usual supersonic wind tunnel. The external acoustic field is generated by a turbulent boundary layer on the wind-tunnel walls. This leads us to the question of the principal possibility of conducting experiments on linear stability theory, since there are no estimates of the admissible level of external disturbances at present. At the same time, a large number of experiments on stability of a supersonic boundary layer was conducted in the T-325 wind tunnel of the Institute of Theoretical and Applied Mechanics of Siberian Division of the Russian Academy of Sciences [3]. With respect to linear instability, these results are in agreement with the theory, although the possible influence of acoustics on the development of instability waves is feared. Therefore, apart from the general theoretical importance, the question of nonlinear interaction of external acoustics and eigen-oscillations in the boundary layer is relevant from the viewpoint of applications, apart from the general theoretical importance, which is related to the possibility of modeling unsteady phenomena. Note that the problem of nonlinear evolution of disturbances in a supersonic boundary layer has attracted attention comparatively recently. A detailed review of the literature can be found in [4].

In the present paper, we consider the interaction of hydrodynamic waves exponentially decaying at infinity and an external acoustic wave within the framework of weakly nonlinear theory. Figure 1 shows experimental data [5] obtained in the T-325 supersonic wind tunnel for M = 2 and dimensionless frequency parameter 10^{-6}–10^{-5} (curves 1–8 show the energy of eigen-oscillations for the following values of the unit Reynolds number: $\mathrm{Re}_1 = 89 \cdot 10^6$, $70 \cdot 10^6$, $48 \cdot 10^6$, $40 \cdot 10^6$, $30 \cdot 10^6$, $19 \cdot 10^6$, $15 \cdot 10^6$, and $6.3 \cdot 10^6$ m^{-1},

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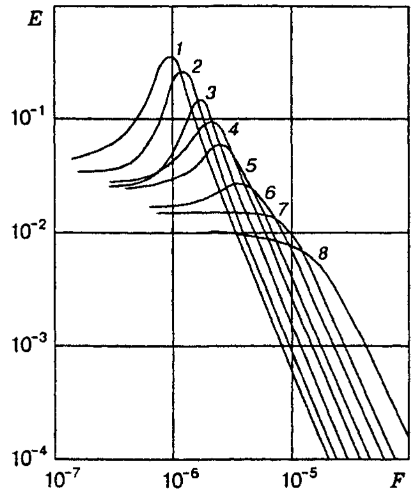

Fig. 1

respectively). Acoustic disturbances with the greatest amplitude are located in the low-frequency range, whereas the frequency of disturbances responsible for the transition is greater by an order of magnitude. The objective of the present work is to determine the degree of influence of weakly nonlinear interaction of the waves at these different frequencies. The method described by Gaponov and Maslennikova [4] is used, and the distinctive feature of the present work is the fact that an acoustic wave, which does not decay at infinity, is used as a pumping wave. Therefore, it is necessary to justify the possibility of using amplitude equations [4] and indicate the method of calculation of the interaction coefficients. The method developed allows one to estimate the admissible level of the acoustic field that does not affect the development of eigen-oscillations in the boundary layer.

Formulation of the Problem. The initial equations for studying the evolution of disturbances in a supersonic boundary layer are the Navier–Stokes equations [4]. The dimensionless parameters of the flow can be represented as the sum

$$Q(x,y,z,t) = Q_b(x,y,z) + \varepsilon Z(x,y,z,t),$$

where Q_b is the solution of steady equations of motion and εZ is the perturbation of the flow parameters ($\varepsilon \ll 1$).

We consider the evolution of disturbances in a supersonic boundary layer on a flat plate at high Reynolds numbers Re_x. In this case, the main flow is independent of the lateral coordinate and weakly depends on x; therefore, we use a parallel flow $Q_b = Q_b(y)$ as an approximation of the main flow. The disturbance evolution is described by a system of nonlinear equations depending on the main flow. In our studies, we impose some additional constraints. In the case of weak nonlinearity, we take into account the contribution of only linear and quadratic terms, and viscosity and heat conductivity are taken into account in linear terms at higher derivatives, which is valid for $\varepsilon \ll 1$. We introduce an eight-component vector-function $\boldsymbol{Z}\,(u, u_y, v, p, T, T_y, w, w_y)$, where u, v, and w are velocity perturbations in the x-, y-, and z-directions, and T and p are perturbations of temperature and pressure; the subscript y denotes the derivative. We write the system of differential equations in the operator form [4] as $\boldsymbol{LZ} = \varepsilon \boldsymbol{M}(q_{ij}, q_{kl})$, where i, $k = 1,\ldots,8$; j and $l = 1,\ldots,4$, $\boldsymbol{q}_i = \boldsymbol{q}_i(z_i, z_{ix}, z_{iz}, z_{it})$ is a four-component vector, the subscripts t, x, and z denote the corresponding derivatives, and L is the linear operator

$$L = A\,\frac{\partial}{\partial t} + B\,\frac{\partial}{\partial x} + C\,\frac{\partial}{\partial z} + D\,\frac{\partial}{\partial y} + E \qquad (1)$$

(A, B, C, D, and E are matrices depending on the main-flow parameters and transport coefficients: viscosity and heat conductivity).

The solution of (1) is constructed by the method of expansion in the small parameter ε and multiscale expansion of the x coordinate, i.e., we introduce a "fast" scale $x_1 = x$ and a "slow" scale X_i, which is possible owing to the large difference between the phase and amplitude variation rates. The scale x_1 is typical of phase variation and X_i is typical of amplitude variation. Taking into account the above said, we assume

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x_1} + \varepsilon \frac{\partial}{\partial X_1} + \varepsilon^2 \frac{\partial}{\partial X_2} + \ldots, \qquad \boldsymbol{Z} = \boldsymbol{Z}^0 + \varepsilon \boldsymbol{Z}^1 + \varepsilon^2 \boldsymbol{Z}^2 + \ldots .$$

In this case, $\boldsymbol{Z}^0$ satisfies the equation $L_0 \boldsymbol{Z}^0 = 0$ or, in expanded form,

$$A \frac{\partial \boldsymbol{Z}^0}{\partial t} + B \frac{\partial \boldsymbol{Z}^0}{\partial x_1} + C \frac{\partial \boldsymbol{Z}^0}{\partial z} + D \frac{\partial \boldsymbol{Z}^0}{\partial y} + E \boldsymbol{Z}^0 = 0. \tag{2}$$

Since the main flow is independent of x_1 ("fast" variable), z, and t, the solution $\boldsymbol{Z}^0$ has the form

$$\boldsymbol{Z}^0 = \mathrm{Real}\Big(\sum_j A_j(X) \boldsymbol{Z}^{0j}(X, y) \exp{(i\theta_j)} \Big). \tag{3}$$

Here $\theta_j = \int \alpha_j(x)\, dx + \beta_j z - \omega_j t$, $A_j(X)$ are constants, the relations for them will be obtained from the next approximation, β_i are the spanwise wavenumbers, and ω_i are their frequencies; the meaning of the wavenumber α_j is explained below.

We consider the interaction of three waves, which satisfy the resonance conditions:

$$\beta_2 + \beta_3 = \beta_1, \qquad \omega_2 + \omega_3 = \omega_1. \tag{4}$$

Substituting (3) into (2), we obtain a system of ordinary differential equations for each $\boldsymbol{Z}^{0j}$:

$$(-i\omega_j A + i\alpha_j B + i\beta_j C + E)\boldsymbol{Z}^{0j} + D \frac{d\boldsymbol{Z}^{0j}}{dy} = 0. \tag{5}$$

The explicit form of the matrices A, B, C, D, and E is not presented here; we only note that they are determined by the parameters of the main steady flow, which depends on the y coordinate and Mach (M) and Reynolds (Re) numbers. They can be found elsewhere, for example, in [6] and other papers on linear stability theory for compressible gas flows, which involves Eqs. (5). The simplest of them are the Dunn–Lin equations for compressible gas or the Orr–Sommerfeld equation for subsonic flow.

In contrast to the case considered in [4], where all the three waves were analogs of the Tollmien–Schlichting waves, here we suggest that one of these waves should degenerate into an acoustic wave at large distances from the surface. Owing to interaction with the boundary layer, the acoustic wave is a superposition of the incident and reflected waves. The problem of linear interaction of a monochromatic wave with the boundary layer is described in detail in [2].

We assume that the subscripts $j = 1$ and 2 correspond to the Tollmien-Schlichting waves and $j = 3$ corresponds to the acoustic wave. The boundary conditions for waves similar to the Tollmien-Schlichting waves have the form

$$Z_1^0 = Z_3^0 = Z_5^0 = Z_7^0 \quad \text{for} \quad y = 0,\, \infty, \tag{6}$$

which corresponds to zero perturbations of velocity (Z_1^0, Z_3^0, Z_7^0) and temperature (Z_5^0) on the surface and at infinity.

For an analog of an acoustic wave, conditions (6) at $y = 0$ remain unchanged, and at $y = \infty$, the corresponding quantities are determined by the parameters of the incident wave; for $y \gg 1$, they have the form

$$\boldsymbol{Z}^{03} = d(\boldsymbol{z}_1^{03} \exp{[i\lambda(y - \delta)]} + q\boldsymbol{z}_2^{03} \exp{[-i\lambda(y - \delta)]}). \tag{7}$$

Here $\boldsymbol{z}_1^{03}$ and $\boldsymbol{z}_2^{03}$ are the constant vectors for the incident and reflected waves, q is the complex reflection factor, δ is the boundary-layer edge, and d is a constant proportional to the strength of the incident wave.

For Tollmien–Schlichting waves, we have the usual eigenvalue problem, from which we determine α_1 and α_2, which enter the expression for the phase θ_j. For an acoustic wave at $y = 0$, the conditions are satisfied for all values of α_3, except for those cases where the reflection factors turn into infinity (see [6]).

In the next approximation, we obtain the following system of equations:

$$L_0 \boldsymbol{Z}^1 = -\sum_{j=1}^{3} \frac{da_j}{dX} B\boldsymbol{Z}^{0j} \exp(i\theta_j) + \boldsymbol{M}(z_k^{0m}, z_p^{0n*})$$

(the asterisk denotes complex conjugation). We clarify the structure of components of the vector $\boldsymbol{M} = \boldsymbol{M}_1 + \boldsymbol{M}_2 + \boldsymbol{M}_3$ obtained from nonlinear terms. The nonlinear terms are determined by the sums of paired products $(z_q^{0k} a_k \exp(i\theta_k) + z_q^{0k*} a_k^* \exp(-i\theta_k^*))(z_p^{0j} a_j \exp(i\theta_j) + z_p^{0j*} a_j^* \exp(-i\theta_j^*))$. We can easily see that the vector $\boldsymbol{M}_1$ proportional to $\exp[i(\beta_1 z - \omega_1 t)]$ is determined by the products $z_q^{02} z_p^{03} a_2 a_3 \exp[i(\theta_2+\theta_3)]$, the vector $\boldsymbol{M}_2$ by the products $z_q^{01} z_p^{03*} a_1 a_3^* \exp[i(\theta_1-\theta_3^*)]$, and the vector $\boldsymbol{M}_3$ by the products $z_q^{01} z_p^{02*} a_1 a_2^* \exp[i(\theta_1-\theta_2^*)]$. If the detunings $\Delta\varphi_1 = \int_0^x (\alpha_2+\alpha_3-\alpha_1)\, dx_1$, $\Delta\varphi_2 = \int_0^x (\alpha_1-\alpha_3^*-\alpha_2)\, dx_1$, and $\Delta\varphi_3 = \int_0^x (\alpha_1-\alpha_2^*-\alpha_3)\, dx_1$ are small, the right side of the previous equation has resonance components relative to the operator L_0 for each trio α_i, β_i, and ω_i. According to [4], phase velocities ω_j/α_j in supersonic flows depend weakly on j; hence, $\Delta\varphi_j$ are small quantities.

Because of degeneration of the operator L_0, a limited solution of system (5) is possible if the right side is orthogonal to the solutions of conjugate problems $\boldsymbol{W}^{0j}$. For Tollmien–Schlichting waves ($j = 1$ and 2), they can be written as

$$\bar{L}_0 \boldsymbol{W}^{0j} = 0, \qquad w_2^{0j} = w_4^{0j} = w_6^{0j} = w_8^{0j} = 0 \quad \text{for} \quad y = 0, \infty. \tag{8}$$

The conjugate problem for an acoustic wave has the form

$$\begin{gathered} \bar{L}_0 \boldsymbol{W}^{03} = 0, \qquad w_2^{03} = w_4^{03} = w_6^{03} = w_8^{03} = 0 \quad \text{for} \quad y = 0, \\ \boldsymbol{W}^{03} = \boldsymbol{w}_1^{03} \exp[i\lambda(y-\delta)] + \bar{q}\boldsymbol{w}_2^{03} \exp[-i\lambda(y-\delta)] \quad \text{for} \quad y \gg 1, \end{gathered} \tag{9}$$

and $\bar{q} = -q$. With account of the direct and conjugate problems, we can write the amplitude equations

$$\frac{da_1}{dX} = k_1 a_2 a_3 \exp(i\Delta\varphi_1), \quad \frac{da_2}{dX} = k_2 a_1 a_3^* \exp(i\Delta\varphi_2), \quad \frac{da_3}{dX} = k_3 a_1 a_2^* \exp(i\Delta\varphi_3), \tag{10}$$

$$k_j = \int_0^\infty (\boldsymbol{M}_k \boldsymbol{W}^{0k})\, dy \Big/ \int_0^\infty (Bz^{0k} \boldsymbol{W}^{0k})\, dy, \qquad j = 1,\ 2.$$

Here a_j are the amplitudes, k_j are the interaction coefficients, $\Delta\varphi_1 = \int_0^x (\alpha_2+\alpha_3-\alpha_1)\, dx_1$, $\Delta\varphi_2 = \int_0^x (\alpha_1-\alpha_3^*-\alpha_2)\, dx_1$, and $\Delta\varphi_3 = \int_0^x (\alpha_1-\alpha_2^*-\alpha_3)\, dx_1$ are the detunings relative to streamwise wavenumbers α_j, $\boldsymbol{W}^{0j}$ is the solution of the problem conjugate to (5) and (6), the vector $\boldsymbol{M}_j$ is determined by nonlinear terms, and B is the matrix depending on the main flow parameters and transport coefficients: viscosity and heat conductivity (see [6]). In the case of parametric development of hydrodynamic waves, the value of the coefficient k_3 is not important, and it was not calculated in this work.

Thus, the procedure of derivation of Eqs. (10) consists of calculation of the amplitudes of three waves, which satisfy the resonance conditions, on the basis of ordinary differential equations (5) with boundary conditions (6) and (7), and functions of conjugate problems (8) and (9).

In the case of parametric development of the Tollmien–Schlichting waves in the field of an acoustic wave, where $|a_1| \ll |a_3|$ and $|a_2| \ll |a_3|$, we can consider that $a_3 \approx \text{const}$, and the disturbance related to the acoustic wave increases in accordance with the linear law, proportionally to $\exp\Big(-\int_0^x \mathrm{Im}(\alpha_1)\, dx\Big)$. Then, the amplification of the amplitude of hydrodynamic waves and the relationship between them at large distances x can be estimated using Eq. (3) and assuming k_1, k_2, $\Delta\varphi_1$, and $\Delta\varphi_2$ to vary weakly as functions of X: $|a_2/a_1| = |\sqrt{k_2/k_1}\,|$, $(1/a_1)(da_1/dX) = \sqrt{k_0}\,|a_3|$, and $(1/a_2)(da_2/dX) = \sqrt{k_0}\,|a_3|$, where $k_0 = k_2 k_1$. The

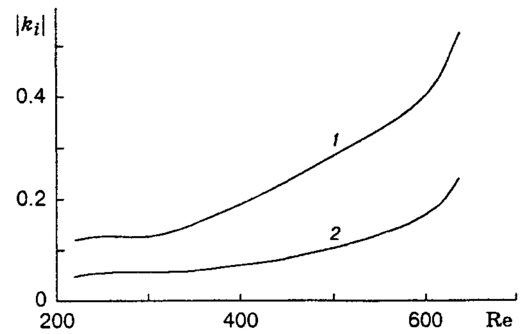

Fig. 2

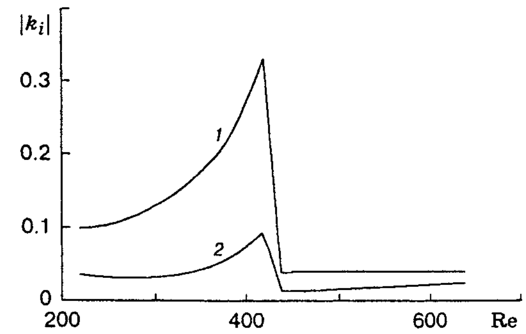

Fig. 3

estimates show that the waves with the greatest real part of the nonlinear growth rate of hydrodynamic waves $\sigma_r = \text{Real}(\sqrt{k_0}) \cdot |a_3|$ grow most intensely at first.

We consider the dependence of σ_r on the inclination of the acoustic wave. Using method [4], we can show that k_0 is an even function of β_3. This means that at $\beta_3 = 0$ the function $k_0(\beta_3)$ takes an extreme value (minimum or maximum). As shown in [6], the following conditions are satisfied for a streamwise acoustic wave and for Tollmien–Schlichting waves:

$$c_{1,2} > 1 - 1/(\text{M} \cos \psi_{1,2}), \qquad c_3 < 1 - 1/\text{M}.$$

Here c_k are the phase velocities and $\psi_i = \arctan(\beta_i/\alpha_i)$ is the angle between the wave and the main-flow direction. This imposes significant constraints on the region of resonance existence.

Calculation Results and Discussion. The coefficients of nonlinear interaction for resonance triplets of waves of the above-discussed type in a supersonic boundary layer on a flat plate were numerically calculated. The steady flow $Q_b(y)$ was self-similar, the stagnation temperature was assumed to be constant and equal to 310 K, which corresponds to the wind-tunnel operation regime, the Prandtl number was Pr = 0.72, and the ratio of specific heats was $\gamma = 1.4$. For each wave mode, the dimensionless frequency parameter $F = 2\pi f \nu_e / U_e^2 = \omega/\text{Re}$ ($\text{Re} = \sqrt{\text{Re}_x}$) remained constant ($f$ is the disturbance frequency and ν_e and U_e are the viscosity and free-stream velocity at the boundary-layer edge, respectively). Normalization was performed to the boundary-layer thickness $\delta = \sqrt{\nu_e x / U_e}$.

The equations were solved by the fourth-order Runge–Kutta scheme, the eigenvalues were sought using the Newton method, and linearly independent solutions were obtained using the orthogonalization procedure [6]. The eigenfunctions of the linear problem for the Tollmien–Schlichting waves were normalized so that $\sup\limits_{0 \leqslant y \leqslant \infty} |Z_3^j(y)| = 1$ ($j = 1, 2$). The functions of the linear problem for an acoustic wave were normalized to the unit streamwise perturbation of the incident wave velocity. The parameters of the acoustic wave and the Tollmien–Schlichting waves were chosen with account of experimental research conducted in the T-325 wind tunnel [7].

The calculations were conducted for M = 2, Re = 220–640, fundamental wave frequency $F_1 = (0.25\text{–}0.90) \cdot 10^{-4}$, acoustic wave frequency $F_3 = (0.447\text{–}0.950) \cdot 10^{-5}$, and angles of the fundamental wave relative to the flow $\psi_1 = 30\text{–}60°$. Spanwise wave numbers satisfied condition (4): $\beta_1 = \beta_2$ and $\beta_3 = 0$. The phase velocity of the acoustic wave was synchronized with the phase velocity of the second Tollmien–Schlichting wave, and the detunings $\Delta\alpha$ remained small everywhere. As a result of the calculations, we determined the fields of interaction coefficients, wavenumbers, velocities, and reflection factors for the acoustic wave as functions of the Reynolds numbers for different frequencies and angles of the triplet relative to the flow at M = 2 on a flat plate.

It follows from the calculations that, for the frequencies of the fundamental wave $F_1 = 0.35 \cdot 10^{-4}$ and acoustic wave $F_3 = 0.047 \cdot 10^{-4}$, there exists a range of angles of the triplet (about 50°) where the growth rate of the interaction coefficients is maximum. Figure 2 shows absolute values of k_i ($i = 1, 2$) as functions of Re for the triplet angle $\psi_1 = 50°$ and the above-mentioned wave frequencies. For other values of F_1 and F_3, the maximum growth rates of the interaction coefficients were obtained for the same angle of the triplet. It is seen from Fig. 2 that the interaction coefficient of the fundamental (first) hydrodynamic wave is approximately twice the interaction coefficient of the second wave. This ratio is also observed for different parameters of the triplet of a given configuration.

The calculations for different F_1 at a fixed frequency of the acoustic wave $F_3 = 0.047 \cdot 10^{-4}$ and angle of the triplet $\psi_1 = 50°$ showed that the maximum values of $k_{1,2}$ are obtained for the frequency $F_1 = 0.5 \cdot 10^{-4}$. The calculation results for these parameters are plotted in Fig. 3 (the notation is the same as in Fig. 2). It follows from the calculation that the phase velocities of the Tollmien–Schlichting waves increase with increasing Re; therefore, any triplet synchronized in phase velocities is destroyed at rather high Re. This instant is seen in the figure as a drastic decrease in the interaction coefficients.

Thus, hydrodynamic waves with a $\sim 50°$ inclination and the dimensionless frequency parameter close to $0.5 \cdot 10^{-4}$ experience the maximum effect of acoustics.

As already noted, all the calculations were conducted for $\beta_3 = 0$. However, the maximum linear effect of acoustics is observed at β_3 other than zero [8]. This question remains open and, hence, further calculations should be performed for $\beta_3 \neq 0$.

Finally, we note that the interaction coefficients in our case are smaller by an order of magnitude than in the case of subharmonic resonance of hydrodynamic waves (see [4]). This fact is related to a large difference in frequency of the external field and Tollmien–Schlichting waves. Since the weakly nonlinear action is proportional to the amplitude of the external acoustic wave, it is negligibly small for low-turbulence wind tunnels (for example, T-325 at the Institute of Theoretical and Applied Mechanics of Siberian Division of the Russian Academy of Sciences). As the Mach number increases, the hydrodynamic and acoustic frequencies should become closer. In addition, the levels of acoustic and induced oscillations inside the boundary layer increase with increasing Mach number [9]. Therefore, the conclusion about the weak effect of acoustics on the degree of amplification of the Tollmien–Schlichting waves cannot be automatically extended to the case of high Mach numbers.

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REFERENCES

1. S. A. Gaponov, "Mathematical simulation of disturbance development in compressible near-wall flows," *Thermophys. Aeromech.*, **1**, No. 1, 75–88 (1994).
2. S. A. Gaponov, "Excitation of Tollmien–Schlichting waves by sound in a supersonic boundary layer," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 3, 59–64 (1983).
3. A. D. Kosinov, N. V. Semionov, S. G. Shevelkov, and O. I. Zinin, "Experiments on the nonlinear instability of supersonic boundary layers," in: D. T. Valentine, S. P. Lin, and W. R. C. Philips (eds.), *Nonlinear Instability of Nonparallel Flows*, Springer-Verlag, Berlin (1994), pp. 196–205.
4. S. A. Gaponov and I. I. Maslennikova, "Subharmonic instability of a supersonic boundary layer," *Teplofiz. Aeromekh.*, **4**, No. 1, 1–10 (1997).
5. V. A. Lebiga, A. A. Maslov, and V. G. Pridanov, "Experimental investigation of the stability of supersonic boundary layer of a flat insulated plate," *Arch. Mech.*, **31**, No. 3, 397–405 (1979).
6. S. A. Gaponov and A. A. Maslov, *Development of Disturbances in Compressible Flows* [in Russian], Nauka, Novosibirsk (1980).
7. A. D. Kosinov, A. A. Maslov, and S. G. Shevelkov, "Experiments on the stability of supersonic laminar-boundary layers," *J. Fluid Mech.*, **219**, 621–623 (1990).
8. S. A. Gaponov and B. V. Smorodsky, "Supersonic boundary layer interaction with streamwise acoustics," in: *Proc. Int. Conf. Methods Aerophys. Res.*, Part 2, Novosibirsk (1996), pp. 70–75.
9. V. A. Lebiga, "Characteristics of oscillations in the test section of a supersonic wind tunnel," in: *Instability of Subsonic and Supersonic Flows* [in Russian], Inst. Theor. Appl. Mech., Sib. Div., Russian Acad. of Sci., Novosibirsk (1982).